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Scattering and Reception by a Flanged Parallel-Plate Waveguide: TE-Mode Analysis

Tah J. Park and Hyo J. Eom

Abstract—**The TE-mode characteristics of scattering and reception by a flanged parallel-plate waveguide are examined. The Fourier transform is used to represent the scattered fields in the spectral domain. The simultaneous equations for the transmitted field coefficients are solved to obtain the solution in an asymptotic series form. Numerical computations are performed to illustrate the behaviors of the scattered field and the transmission coefficients versus the aperture size.**

I. Introduction

Electromagnetic scattering from a conducting double-wedge has been extensively studied with asymptotic high-frequency techniques [1], [2] since an exact closed-form solutionis still unknown. TM-mode scattering from a flanged parallel-plate waveguide (a special double-wedge geometry) was considered in [3] using the Weber–Schafheitlin integral technique. In this paper, we examine TE-mode scattering from the flanged waveguide by utilizing the Fourier transform and the mode-matching technique [4], [5]. In the next section, we present the scattered field as an asymptotic series which can be represented in closed form in high-frequency limit. Numerical computations are presented to illustrate the behaviors of the scattered field and the transmission coefficient. A brief summary of the theoretical development is given.

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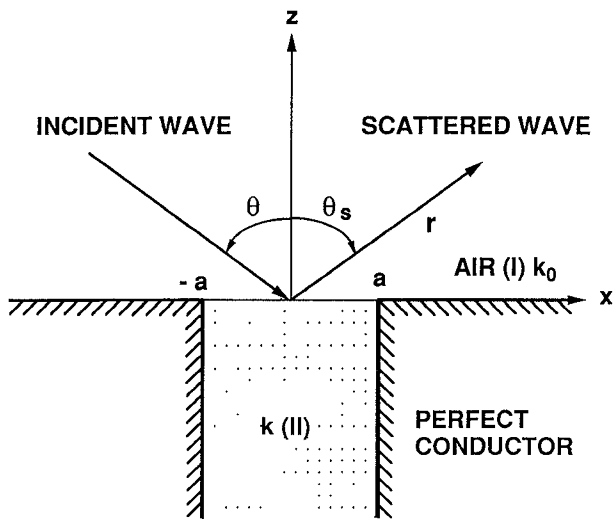

Fig. 1.

II. Scattered and Received Fields Derivation

Fig. 1 shows a perfect-conducting, flanged, parallel-plate waveguide of width $2a$. In Region I ($z > 0$) an incident field E_y^i (TE mode: transverse-electric-to-propagation-direction) impinges on the flanged parallel-plate waveguide. Region II ($z < 0$, $-a < x < a$) denotes the waveguide interior. The wave numbers of Regions I and II are $k_o(= 2\pi/\lambda)$ and k, respectively, and the time factor $e^{-j\omega t}$ is suppressed.

In Region I the total electric field consists of the incident, reflected, and scattered fields which are written as

$$E_y^i(x,z) = e^{jk_x x - jk_z z}$$
$$E_y^r(x,z) = -e^{jk_x x + jk_z z}$$
$$E_y^s(x,z) = 1/(2\pi)\int_{-\infty}^{\infty} \hat{E}_y^s(\zeta) e^{-j\zeta x + jk_1 z}\, d\zeta$$

where

$$k_x = k_0 \sin\theta$$
$$k_z = k_0 \cos\theta$$
$$k_1 = \sqrt{k_0^2 - \zeta^2}$$
$$\tilde{E}_y^s(\zeta) = \int_{-\infty}^{\infty} E_y^s(x,0) e^{j\zeta x}\, dx .$$

Since $H_x(x,z) = -1/(j\omega\mu)\partial E_y(x,z)/\partial z$, the corresponding x components of the incident, reflected, and scattered H fields may be readily obtained.

In Region II the total transmitted field may be represented as

$$E_y^t(x,z) = \sum_{m=1}^{\infty} d_m \sin a_m(x+a) e^{-j\xi_m z} \tag{1}$$

where

$$a_m = m\pi/(2a)$$
$$\xi_m = \sqrt{k^2 - a_m^2} .$$

To determine the unknown coefficients d_m it is necessary to enforce continuity in the tangential E and H fields. First, continuity of the tangential E field along the x–axis yields

$$E_y^s(x,0) = E_y^t(x,0) \quad |x| < a$$
$$= 0 \quad |x| > a .$$

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Taking the Fourier transform of both sides of the above equation, we get

$$\tilde{E}_y^s(\zeta) = \int_{-\infty}^{\infty} E_y^s(x,0)e^{j\zeta x}\,dx = \int_{-a}^{a} E_y^t(x,0)e^{j\zeta x}\,dx. \quad (2)$$

Substituting (1) into (2) and performing the integration with respect to x, we obtain

$$\tilde{E}_y^s(\zeta) = \sum_{m=1}^{\infty} d_m \frac{a_m}{(\zeta^2 - a_m^2)}\left[e^{j\zeta a}(-1)^m - e^{-j\zeta a}\right] \quad (3)$$

Second, continuity of the tangential H field along $-a < x < a$, $z = 0$, gives

$$H_x^i(x,0) + H_x^r(x,0) + H_x^s(x,0) = H_x^t(x,0)$$

$$2k_z e^{jk_x x} - \int_{-\infty}^{\infty} \frac{k_1}{2\pi}\tilde{E}_y^s(\zeta)e^{-j\zeta x}\,d\zeta = \sum_{m=1}^{\infty} d_m\,\xi_m \cdot \sin a_m(x+a). \quad (4)$$

Substituting (3) into (4), we obtain

$$2k_z e^{jk_x x} - \frac{1}{2\pi}\sum_{m=1}^{\infty} d_m\,a_m \int_{-\infty}^{\infty} \cdot \frac{(-1)^m e^{-jsa} - e^{-jsa}}{\zeta^2 - a_m^2} k_1 e^{-j\zeta x}\,d\zeta = \sum_{m=1}^{\infty} d_m\,\xi_m \sin a_m(x+a).$$

In order to determine the coefficient d_m, we multiply the above equation by $\sin a_n(x+a)$ and integrate both sides with respect to x from $-a$ to a. We then obtain

$$\frac{2k_z a_n}{a_n^2 - k_x^2}\left[-(-1)^n e^{jk_x a} + e^{-jk_x a}\right] = \frac{1}{2\pi}\sum_{m=1}^{\infty} \cdot d_m I_{mn} + d_n \xi_n a \quad (5)$$

where

$$I_{mn} = \int_{-\infty}^{\infty} \cdot \frac{a_m a_n\left[(-1)^m e^{j\zeta a} - e^{-j\zeta a}\right]\left[(-1)^n e^{-j\zeta a} - e^{j\zeta a}\right]k_1}{(\zeta^2 - a_m^2)(\zeta^2 - a_n^2)}\,d\zeta.$$

Contour integral evaluation of I_{mn} may be performed in the complex ζ plane to give

$$I_{mn} = 2\pi a\eta_m\delta_{mn} - (I_{1mn} + I_{2mn}) \quad (6)$$

where $\eta_m = \sqrt{k_0^2 - a_m^2}$ and δ_{mn} is Kronecker delta. The explicit expressions for I_{1mn} and I_{2mn} are given in [5]. We find

$$I_{1mn} = \int_0^{\infty} \frac{-4j\alpha\beta(-1)^n e^{2jk_0 a}e^{-2k_0 a\nu}\sqrt{\nu(-2j+\nu)}}{\left[(1+j\nu)^2 - \alpha^2\right]\left[(1+j\nu)^2 - \beta^2\right]}\,d\nu \quad (7)$$

$$I_{2mn} = \int_0^{\infty} \frac{4j\alpha\beta\sqrt{\nu(-2j+\nu)}}{\left[(1+j\nu)^2 - \alpha^2\right]\left[(1+j\nu)^2 - \beta^2\right]}\,d\nu$$

where

$$\alpha = a_m/k_0, \quad \beta = a_n/k_0.$$

Performing the integrations with respect to v [5], we obtain

$$I_{1mn} = -\frac{2\alpha\beta e^{2jk_0 a}(-1)^n}{(\alpha^2 - \beta^2)} \cdot \sum_{l=1}^{\infty} S_l[[A(t_1) - A(t_2)]/\alpha - [A(t_3) - A(t_4)]/\beta] \quad (8)$$

$$I_{2mn} = \frac{4j\alpha\beta}{(\alpha^2 - \beta^2)}\left[\frac{\sqrt{1-\alpha^2}}{\alpha}\sin^{-1}\alpha - \frac{\sqrt{1-\beta^2}}{\beta}\sin^{-1}\beta\right]$$

where

$$S_l = \binom{0.5}{l-1}(0.5j)^{l-1\,5}$$

$$A(t) = (-1)^l \pi t^{l-0.5} e^{pt} erfc(\sqrt{pt}) + 2^{1-l}\sqrt{\pi}p^{0.5-l} \cdot \sum_{r=0}^{t-1}(2l-2r-3)!!(-2pt)^r$$

$$p = 2k_0 a$$

$erfc(\cdots)$: complementary error function

$$t_1 = (\alpha - 1)j, \qquad t_2 = (-\alpha - 1)j,$$
$$t_3 = (\beta - 1)j, \qquad t_4 = (-\beta - 1)j.$$

Note that I_{1mn} in (8) is expressed in terms of an asymptotic series of which the lth term is $O\left(1/(k_0 a)^{l-0.5}\right)$. This series expression for I_{1mn} converges only for $|2k_0 a/(m\pi)| > 1$; hence, it is computationally more efficient to use the rapidly convergent integral (7) than (8) for the evaluation of I_{1mn}. When $k_0 a \to \infty$ the branch-cut contribution becomes negligible and $I_{mn} \to 2\pi a\eta_m\delta_{mn}$.

Substituting I_{mn} of (6) into (5) and solving for d_m, we obtain

$$D = (U - R)^{-1}S = S + RS + R^2S + \cdots \quad (9)$$

where D is a column matrix with elements d_m, U is the identity matrix, R is a full matrix with elements r_{nm} and S is a column matrix with elements s_n. The explicit expressions for r_{nm} and s_n are as follows:

$$r_{nm} = \frac{(I_{1mn} + I_{2mn})}{2\pi(\xi_n + \eta_n)a}$$

$$s_n = \frac{2k_z a_n\left[-(-1)^n e^{jk_x a} + e^{-jk_x a}\right]}{(\xi_n + \eta_n)a(a_n^2 - k_x^2)}.$$

If $k = k_o$, then

$$r_{nm} = \frac{(I_{1mn} + I_{2mn})}{4\pi\xi_n a}$$

$$s_n = \frac{k_z a_n\left[-(-1)^n e^{jk_x a} + e^{-jk_x a}\right]}{\xi_n a(a_n^2 - k_x^2)}.$$

An examination of r_{nm} reveals that $r_{nm} \sim O\left[1/\sqrt{k_0 a}\right]$ for $k_0 a > 1$ and $\xi_n + \eta_n \neq 0$. For $k_0 a \gg 1$ the branch-cut contribution may be ignored ($r_{nm} \approx 0$). Thus (9) reduces to the Kirchhoff approximation

$$d_m \approx s_m. \quad (10)$$

The branch-cut contributions I_{1mn} and I_{2mn} in I_{mn} account for coupling between the continuous spectrum of $E_y^s(x,z)$ and the discrete spectrum of $E_y^t(x,z)$. When $k_o a \gg 1$, the magnetic current at the aperture, $E_y^t(x,0)$, is approximately given as $E_y^i(x,0)$ which has a very narrow spectral width; hence, the branch-cut contributions can be ignored.

Another special case of interest is low-frequency scattering ($k_0 a \ll 1$). When $k_o a \ll 1$, the dominant element among r_{nm} is r_{11} whose value is approximately given by $2/\pi^2$. Hence, we have

$$d_1 \approx s_1/(1 - r_{11}). \quad (11)$$

III. Numerical Computations

The time-averaged power density P, which is received by the flanged parallel-plate waveguide, is

$$P = \frac{1}{2}\int_{-a}^{a} \mathrm{Re}\left(\bar{E}^t \times \bar{H}^{t*}\right)\cdot(-\hat{z})\,dx = \frac{a}{2\omega\epsilon}\sum_{m=1}^{\infty}\mathrm{Re}\left(\xi_m^*\right)|d_m|^2$$

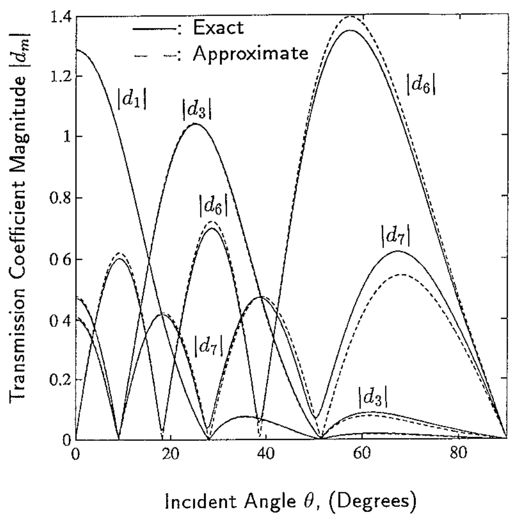

Fig. 2.

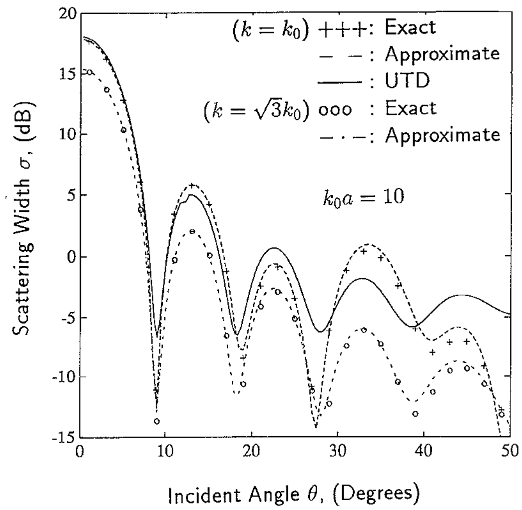

Fig. 3.

where the $\bar{E}^t$ and $\bar{H}^t$ are the transmitted E and H field vectors, and the symbols $\mathrm{Re}(\cdots)$ and $(\cdots)^*$ denote, respectively, the real part of $(\cdots)$ and the complex conjugate of $(\cdots)$.

The far-zone scattered field at a distance r from the origin can be evaluated by utilizing the stationary phase approximation. We find

$$E_y^s(\theta_s, \theta) = e^{j(k_0 r - \pi/4)} \sqrt{\frac{k_0}{2\pi r}} \cos\theta_s \cdot \sum_{m=1}^{\infty} d_m a_m \frac{e^{-jk_0 a \sin\theta_s}(-1)^m - e^{jk_0 a \sin\theta_s}}{(k_0 \sin\theta_s)^2 - a_m^2} \qquad (12)$$

where $\theta_s = \sin^{-1}(x/r)$ and $r = \sqrt{x^2 + z^2}$.

We first evaluate the scattered field for low-frequency scattering ($k_o a \ll 1$). Substituting (11) into (12), and taking the leading term ($m = 1$), we obtain

$$E_y^s(\theta_s, \theta) \approx 0.5(k_o a)^2 \frac{e^{j(k_0 r - 3\pi/4)}}{\sqrt{k_0 r}} \cos\theta \cos\theta_s. \qquad (13)$$

Note that (13) agrees well with other low-frequency solution of scattering from a narrow groove [6].

In Table I the transmission coefficients d_m are tabulated versus $2a/\lambda$ for $\theta = 0°$. Note that $d_2 = d_4 = \cdots = 0$ because $\theta = 0°$.

In Fig. 2 $|d_m|$ are plotted versus θ for $k_0 a = 10$ ($k = k_0$). Both (9) and (10) are used to obtain the exact and approximate solutions respectively. Fig. 2 shows that the exact and approximate solutions agree well for high-frequency scattering.

Fig. 3 show the backscattered σ ($\theta_s = -\theta$) versus θ for $k_0 a = 10$ where $\sigma = \lim_{r\to\infty} 2\pi r \left|E_y^s(\theta_s, \theta)/E_y^i(\theta)\right|^2$. The number of coefficients d_m used in the computation are 10. Comparison of the exact and approximate solutions for the cases $k = k_0$ and $\sqrt{3}k_0$ shows that an increase in k results in a decrease in σ. In Fig. 3 the exact solution is compared with the UTD solution which may be obtained by superimposing the singly-diffracted solutions [2]. This comparison between the UTD solution and ours indicates good agreement (less than 2 dB error) when $\theta < 20°$.

IV. Concluding Remarks

Using the Fourier transform and mode-matching approach, we obtain the series solution to scattering from the flanged waveguide. Numerical computations are performed to illustrate the behaviors of the fields scattered by and received by the flanged-parallel plate waveguide. The series solution, which is based on (9), is exact and numerically efficient.

TABLE I

	Amplitude of d_m			Phase, degrees		
$2a/\lambda$	d_1	d_3	d_5	d_1	d_3	d_5
0.51	3.0714	0.1710	0.0738	−19.61	−41.26	−37.79
0.61	2.0517	0.1506	0.0592	−7.27	−63.11	−55.66
0.71	1.7721	0.1771	0.0644	−3.26	−76.02	−68.30
0.81	1.6238	0.2210	0.0758	−1.27	−83.70	−76.61
0.91	1.5304	0.2833	0.0910	−0.20	−87.72	−81.45
1.01	1.4461	0.3635	0.1092	0.36	−89.03	−83.66
1.11	1.4194	0.4684	0.1299	0.61	−88.06	−83.80
1.21	1.3841	0.6088	0.1524	0.65	−84.83	−82.08
1.31	1.3565	0.8061	0.1754	0.53	−78.73	−78.34
1.41	1.3344	1.1150	0.1947	0.22	−67.75	−71.60
1.51	1.3262	1.5705	0.1600	−0.83	−27.14	−59.36
1.61	1.3327	1.0477	0.1595	−0.55	−12.05	−75.68

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